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Failure in Complex Social Networks

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A class of inhomogenously wired networks called "scale-free" networks have been shown to be more robust against failure than more homogenously connected exponential networks. The robustness of scale-free networks consists in their ability to remain connected even when failure occurs. The diffusion of information and disease across a network only requires a single contact between nodes, making network connectivity the crucial determinant of whether or not these "simple contagions" will spread. However, for "complex contagions," such as social movements, collective behaviors, and cultural and social norms, multiple reinforcing ties are needed to support the spread of a behavior diffusion. I show that scale-free networks are much less robust than exponential networks for the spread of complex contagions, which highlights the value of more homogenously distributed social networks for the robust transmission of collective behavior.

Keywords: collective dynamics, complex contagions, diffusion, network robustness, scale-free networks, social dynamics, social networks

Tolerance against failures and errors is an important feature of many complex networked systems (Albert et al., 2000; Hartwell et al., 1999; Holme and Kim, 2002). It has been shown that a class of inhomogeneously wired networks called "scale-free" (Albert et al., 2000; Barabasi and Albert, 1999; Barabasi et al., 1999) networks can be surprisingly robust to failures, suggesting that socially self-organized systems such as the World Wide Web (Huberman and Adamic, 1999), the Internet (Cohen et al., 2000), and other kinds of social networks (Redner, 1998) may have significant tolerance against failures by virtue of their scale-free degree distribution. I show that this finding only

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holds on the assumption that the diffusion process supported by the network is a simple one, requiring only a single contact in order for transmission to be successful. For complex contagions (Centola et al., 2007; Centola and Macy, 2007), such as the spread of cultural norms (Axelrod 1997), collective behavior (Granovetter, 1978) or cooperation (Nowak and May, 1992; Riolo et al., 2001), multiple sources of reinforcement are needed for transmission to be successful (Centola and Macy, 2007; McAdam and Paulsen, 1993). I find that on networks with high levels of local clustering, as is typical of social networks (Newman and Park, 2003; Watts and Strogatz, 1998; Keeling, 1999), a scale-free degree distribution makes the social topology much more sensitive to failure due to accidents and errors than having a more homogeneous, exponential degree distribution.

Scale-free networks are characterized by a power law decay in the degree distribution, such that $P(k) \approx k^g$, which gives some nodes very large degrees while most others have very small degrees. The Web and the Internet have been shown to have scale-free properties (Albert et al., 2000; Barabasi and Albert, 1999); however, many examples of social networks, such as scientific collaboration networks (Newman 2001) and friendship networks (Newman, 2003), are frequently not scale-free but do have right-skewed distributions with exponential decay (Newman, 2003; Strogatz, 2001). Networks with exponential degree distributions $P(k) \approx \exp(-\frac{k}{c})$ provide moderate "hubbiness" while also having exponential decay for large k.

Exponential and scale-free networks are compared by using networks of the same size ($N = 10{,}000$), average degree ($<k> = 4$), and level of clustering ($CC = .25$) (Newman and Park, 2003; Watts and Strogatz, 1998; Klemm and Eguiluz, 2002). Error tolerance is tested by randomly removing a fraction, *f*, of nodes from the network (Albert et al., 2000) and then measuring the average size of cascades, S (the number of nodes reached by a contagion), which originate from a randomly chosen seed neighborhood. Cascades of complex contagions are measured by assigning each node a threshold of adoption ($t = 2/k$), such that each node must have at least two neighbors activated in order to become activated (Centola and Macy, 2007). A randomly chosen "seed" neighborhood (one individual node and all of its neighbors) is then activated to initiate the cascade dynamics.[1] This process is repeated over 1,000 realizations to produce an ensemble average cascade size, $<S>$, for each value of *f*.

[1]Asynchronous updating with random order and without replacement eliminates potential order effects and guarantees that every node is updated within a round of decision-making, which is defined as *N* time-steps.

In order for complex contagions to propagate across a social network, the network not only must remain connected but must also have sufficient local structure in the form of connected clusters with triadic closure to support social reinforcement from one cluster

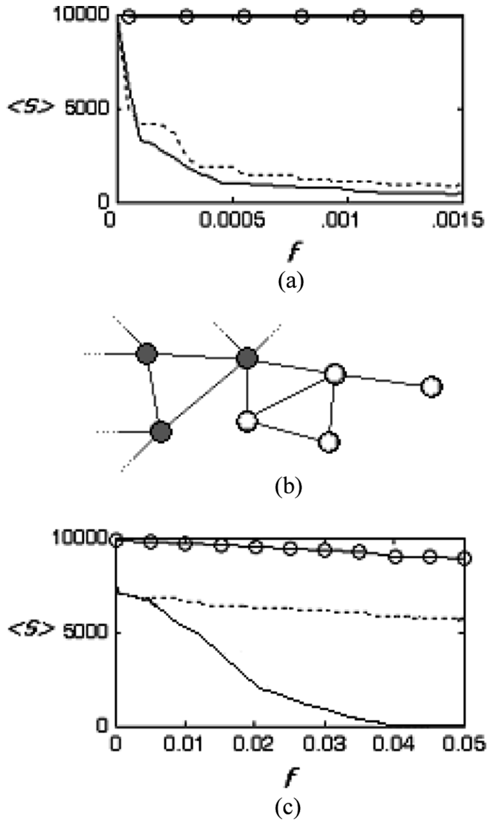

FIGURE 1 Spread of simple and complex contagions on scale-free and exponential networks. (a) Average size of cascades of complex contagions as fraction *f* of nodes are removed from a clustered scale-free network by random failure (dotted line) or targeted attack (solid line), and the connectedness of the network (solid line with circles) as the size of the largest connected component. (b) Minimally complex contagions require that each node have 2 neighbors activated in order to become activated. This can cause "bottlenecks" where a contagion cannot spread from activated nodes (shown in gray) to reach other nodes in a connected network (shown in white). (c) Average size of cascades of complex contagions on an exponential network with random failure (dotted line) and targeted attack (solid line), and size of the largest connected component (solid line with circles).

to the next (Centola and Macy, 2007). When a scale-free network suffers node attrition (Fig. 1a), it quickly reaches a critical fraction of removed nodes, $f_c \sim .0002$, above which cascades can only reach less than half of the network. For slightly higher values of f, cascades fail to spread beyond the immediate region of the seed neighborhood ($<S> < 1000$). This is independent of whether nodes are removed by targeted attack (removing the most connected nodes first, solid line in Fig. 1a) or by random failure (dotted line in Fig. 1a). At the critical transition f_c, the scale-free network remains connected, as shown by the average size of cascades of simple contagions, $<S> \approx N$. However, complex contagions exhibit sensitivity to failure because small breaks in the network reduce crucial overlap between the minimally connected neighborhoods, resulting in the inability of multiple signals to pass between them (Centola et al., 2007). This weakness is endemic to clustered scale-free networks because of the large fraction of the population with minimal connectivity.

The exponential network has more nodes with moderate degree; thus, there are many redundant pathways for local reinforcement, making the network much less sensitive to failure. The formation of "bottlenecks" (illustrated in Fig. 1b) limits complex contagions to reaching only 70% entire network even with zero failure (shown in Fig. 1c). Despite this, even with 5% random failure ($f = .05$, dotted line in Fig. 1c) cascades on exponential networks can still reach more than 50% of the network. Targeted attacks (solid line in Fig. 1c) have a much greater impact on the exponential network, eventually causing cascades sizes to drop to zero; however, exponential networks do not have a critical transition for complex contagions and are robust to targeted attacks up to losing the 100 most connected nodes (1%, $f = .01$). This robustness of the exponential degree distribution for the diffusion of complex contagions may explain its relative abundance in social networks, which are powerful pathways for cultural transmission and social reinforcement despite continual attrition due to death and mobility.

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